

AFTERGLOW LIGHT CURVES AND BROKEN POWER LAWS: A STATISTICAL STUDY

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ABSTRACT

In gamma-ray burst research it is quite common to fit the afterglow light curves with a broken power law to interpret the data. We apply this method to a computer simulated population of afterglows and find systematic differences between the known model parameters of the population and the ones derived from the power law fits. In general, the slope of the electron energy distribution is overestimated from the pre-break light curve slope while being underestimated from the post-break slope. We also find that the jet opening angle derived from the fits is overestimated in narrow jets and underestimated in wider ones. Results from fitting afterglow light curves with broken power laws must therefore be interpreted with caution since the uncertainties in the derived parameters might be larger than estimated from the fit. This may have implications for Hubble diagrams constructed using gamma-ray burst data.

Subject headings: gamma rays: bursts — gamma rays: theory — methods: data analysis

1. INTRODUCTION

Many gamma-ray burst (GRB) afterglows can be interpreted within the standard fireball model (see Piran 2005, for a recent review), where a jet structure is implied from the steepening of the light curve (Rhoads 1999). By fitting the light curves with a (sharp or smoothly joined) broken power law (see e.g. Beuermann et al. 1999), it is possible to determine the time of the steepening (the so called jet break time) and the pre-break and post-break slopes of the light curves. This data can then be used to obtain information about the parameters of the underlying model, e.g. the jet opening angle which can then be used to estimate the energy of the burst (e.g. Frail et al. 2001; Ghirlanda et al. 2004; Friedman & Bloom 2005).

Although it has been shown that the afterglow light curves in the standard model can be approximated with a broken power law (e.g. Sari et al. 1998; Rhoads 1999), the accuracy of the model parameters derived in that way has not been tested. In this letter we show that the results from fitting a broken power law to afterglow light curves must be interpreted with caution as the parameters derived from the fit can be systematically different from the actual model parameters. We do this by creating a population of simulated afterglow light curves using our version of the standard model (Jóhannesson et al. 2005) and then fit them with a broken power law. In section 2 we shortly describe the model and procedure used, present the results and then conclude in section 3 with a discussion of our findings.

2. THE MODEL AND RESULTS

We used the instantaneous energy injection version of the standard model presented in Jóhannesson et al. (2005). This is an extension of the Rhoads (1999) model, and includes a more detailed calculation of the synchrotron emission and the effects of the equal arrival time surface (EATS). We created a population of 20,000 afterglow light curves where the model parameter val-

ues were selected at random from a normal distribution over a narrow parameter range given in table 1. We assume a constant density interstellar medium for all events. The standard deviation of the distribution for each parameter was fixed at 1/4 of its range and the distribution was clipped to fit within each parameter range. The range of parameters was chosen after fitting several afterglows with our model over a wide range of frequencies (see examples in Jóhannesson et al. 2005; de Ugarte Postigo et al. 2005). Although the fraction of energy contained in electrons, ϵ_e , and magnetic field, ϵ_B , do not directly enter our formalism, they are included as they can affect the results via the characteristic synchrotron frequencies, ν_m and ν_c (Sari et al. 1998). The viewing angle of the observer, θ_v , can smooth the jet break (Granot et al. 2001) and was also included as a parameter. The light curves were evenly sampled in the logarithm of observer time and consisted of about 35 points each (see fig. 1 for a typical example). To make the synthetic data more realistic, we added normally distributed random fluctuations with a standard deviation of 3%. We also assumed a fixed error of 5% for each data point. We find that our results are not sensitive to the values of these error parameters and lowering the error estimate only reduces the number of successful fits without reducing the scatter in our results. The results were also tested for the effects of the EATS by turning it off in a test sample. We found that it had no significant effect.

According to standard fireball theory, the simplest afterglow light curves can be described with the widely adopted analytical approximation (Rhoads 1999),

$$F_\nu(t) \propto \begin{cases} t^{-3(p-1)/4} & t < t_j, \\ t^{-p} & t > t_j, \end{cases} \quad (1)$$

for a given observing frequency, ν , in the range $\nu_m < \nu < \nu_c$. Here t is the time from the onset of the burst in the observer's frame, t_j is the jet break time in the observer's frame and p is the electron energy distribution index. Since the jet break is in general not sharp, eq. (1) is often replaced with a smoothly joined broken power

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TABLE 1
THE PARAMETERS OF THE AFTERGLOW POPULATION.

Parameter	Min Value	Max Value
$E_0/10^{51}$ ergs	0.5	2
Γ_0	100	1000
n_0 cm $^{-3}$	0.1	10
θ_0	1 $^\circ$	20 $^\circ$
p	2	2.6
ϵ_e	0.1	0.5
$\epsilon_B/10^{-4}$	0.5	20
θ_v/θ_0	0	1
z	0.1	6

The parameters in the afterglow population are normally distributed over the ranges shown here. The parameters are all distributed logarithmically, except p , θ_v/θ_0 and z . See text for more detail.

law (Beuermann et al. 1999),

$$F_\nu(t) = F_j \left[\left(\frac{t}{t_j} \right)^{\alpha_1 n} + \left(\frac{t}{t_j} \right)^{\alpha_2 n} \right]^{-1/n}. \quad (2)$$

Here, F_j is the flux at t_j , $-\alpha_1$ and $-\alpha_2$ are respectively the pre-break and post-break light curve slopes and n is a numerical factor controlling the sharpness of the break.

The light curve break occurs when the jet enters the sideways expanding regime (called exponential regime in Rhoads 1999) and the observer receives light from the entire jet surface. The break time can be approximated as that point in time when $\theta \approx 1/\Gamma$, where θ is the opening angle of the jet and Γ is the Lorentz factor of the relativistically moving shock front. Using analytical approximations from Jóhannesson et al. (2005) we find that

$$t_j \approx 1.21(1+z) \left(\frac{E_0/10^{51} \text{ ergs}}{n_0/1 \text{ cm}^{-3}} \right)^{1/3} \left(\frac{\theta_0}{0.1} \right)^2 \text{ days}, \quad (3)$$

where E_0 is the total energy injected into the jet, n_0 is the constant interstellar medium particle density, θ_0 is the initial opening angle of the jet in radians and z is the redshift of the burst. This formulation differs slightly from the one given in Rhoads (1999), because we choose to use the energy injected into the jet rather than the isotropic equivalent energy in order to better isolate θ_0 in the equation.

As is often done when fitting GRB afterglows (e.g. Wijers et al. 1997; Beuermann et al. 1999; Zeh et al. 2005), we fitted the synthetic R-band light curves with both a sharply broken power law as in equation (1) and a smoothly joined broken power law as in equation (2). These will hereafter be referred to as sharp fits and smooth fits, respectively. The Levenberg-Marquardt method of Press et al. (1996) was used to minimize the χ^2 value of the fit in both cases. Although the starting point of the fitting procedure was chosen as the theoretically correct values of t_j , α_1 , α_2 and F_j , it did not result in an acceptable fit in every case. Only those events where the χ^2 per degree of freedom is less than 1 for the smooth fits and 2 for the sharp fits were selected for further study. A higher threshold was used for the sharp fits since the numerically generated light curves are very smooth and not well represented by a sharply broken power law (see figure 1). About half of the fits fulfilled those requirements in both cases. Since GRB afterglow measurements normally extend from a few hours after

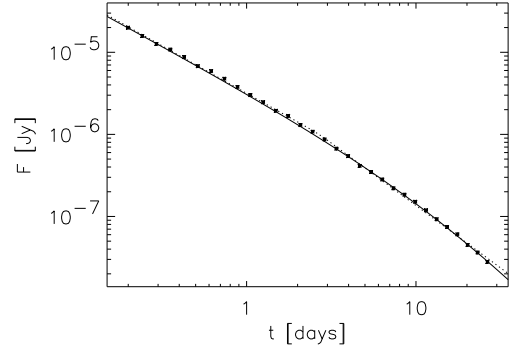


FIG. 1.— A typical light curve from our sample. The points are from our model calculations, the solid curve shows the smooth fit and the dotted curve the sharp fit. The light curve break time determined by the smooth fit is rather accurate in this case, around 22 days, whereas the sharp fit break time is around 3 days. The light curve slopes agree well with the data for the smooth fit but the post-break slope in the sharp fit is too shallow as may be expected from the break time error.

TABLE 2
THE PARAMETERS OF THE GAUSSIAN FITS TO THE ERROR DISTRIBUTIONS.

	Smooth		Sharp	
	Mean	Std. dev.	Mean	Std. dev.
$\Delta p_1/p$	0.097	0.042	0.13	0.042
$\Delta p_2/p$	-0.18	0.064	-0.29	0.068
$\Delta \theta_j/\theta_0$	-0.034	0.091*	-0.19	0.28**

*The Gaussian fit is not particularly good here and the distribution is actually wider.

**There is a narrow peak around 0 on top of the Gaussian.

the burst to about a month, we limited our data and t_j to this time range in the fitting procedure. Using equation (3), it can be shown that this range also limits the range of derivable opening angles to about 1.5 $^\circ$ - 15 $^\circ$, depending on other burst parameters. With real data, this range of opening angles can be further reduced if there is a bright underlying host or a supernova component making the afterglow light difficult to observe.

The results for t_j from the fitting procedure can be used to find an estimate of the opening angle of the jet by inverting equation (3),

$$\theta_j = \left(\frac{t_j/1 \text{ day}}{121(1+z)} \right)^{1/2} \left(\frac{n_0/1 \text{ cm}^{-3}}{E_0/10^{51} \text{ ergs}} \right)^{1/6}. \quad (4)$$

Here, the opening angle is denoted by θ_j to distinguish it from the known initial opening angle of the burst, θ_0 , from our numerical model calculation, although theoretically they should be equal. The results for the derived slopes, α_1 and α_2 , can be used to find the value of p and equation (1) gives two different values, $p_1 = (4/3)\alpha_1 + 1$ and $p_2 = \alpha_2$. Comparison of the values obtained this way from the fitting procedure to the parameters known from the numerical model is shown in figure 2 as the distribution of relative differences between the derived parameters and the known model parameters: $\Delta p_1/p$, $\Delta p_2/p$ and $\Delta \theta_j/\theta_0$, where $\Delta p_1 = p_1 - p$, $\Delta p_2 = p_2 - p$ and $\Delta \theta_j = \theta_j - \theta_0$. Those distributions were each fitted with a Gaussian profile that is overlaid on the distributions in

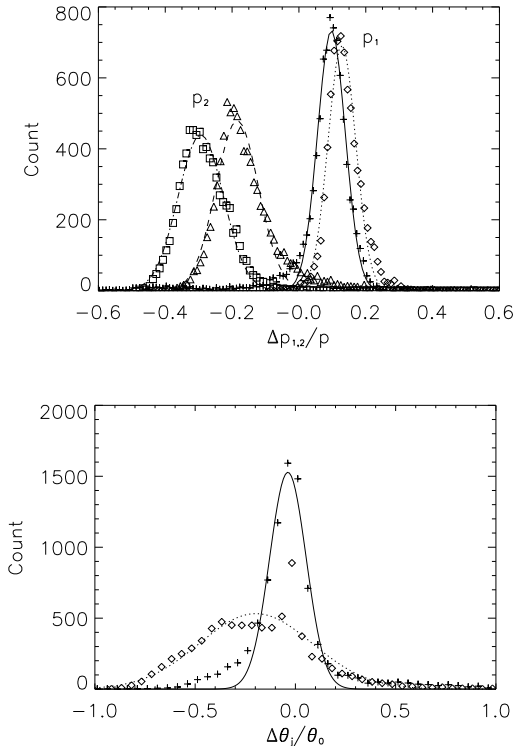


FIG. 2.— The distribution of relative differences in the value of parameters deduced from the fits. The upper panel shows the distribution in $\Delta p_1/p$ and $\Delta p_2/p$. The smooth and sharp fits are represented, respectively, by plus-signs and diamonds for $\Delta p_1/p$ and triangles and boxes for $\Delta p_2/p$. The lower panel shows the distribution in $\Delta\theta_j/\theta_0$ for smooth (plus-signs) and sharp (diamonds) fits. The overlaid curves are Gaussian fits to the corresponding distributions. The parameters of the Gaussians are given in table 2. The count is higher in the lower panel because the binsize is larger.

the figure. The parameters of the Gaussian profiles are presented in table 2.

The distributions and the corresponding Gaussian fits clearly show a systematic difference in the evaluation of p , where p_1 is overestimated and p_2 underestimated. From the $\Delta\theta_j/\theta_0$ distribution, it is clear that the sharply broken power law does not do a good job in determining the opening angle and there is both a significant underestimate and a large standard deviation. This is also reflected in the $\Delta p_2/p$ distribution since an underestimate of the opening angle will make p_2 smaller. It should however be noted that there is a high narrow peak around zero in the $\Delta\theta_j/\theta_0$ distribution for the sharp fits which is not included in the Gaussian fit. The overall standard deviations in the Gaussian fits are not very high but should be considered when using θ_j to correct the isotropic energy, since the correction factor is approximately proportional to θ_j^2 .

We have checked for correlations between the parameters p_1 , p_2 and θ_j deduced from the fits and the known parameters of our numerical model, which range is shown in table 1. We find no significant correlation within that parameter range, except between θ_0 and $\Delta\theta_j/\theta_0$. Figure 3 shows a scatter plot of $\Delta\theta_j/\theta_0$ as a function of θ_0 for both smooth and sharp fits and also a binned average shown as a solid line. It is clear that narrow jet opening

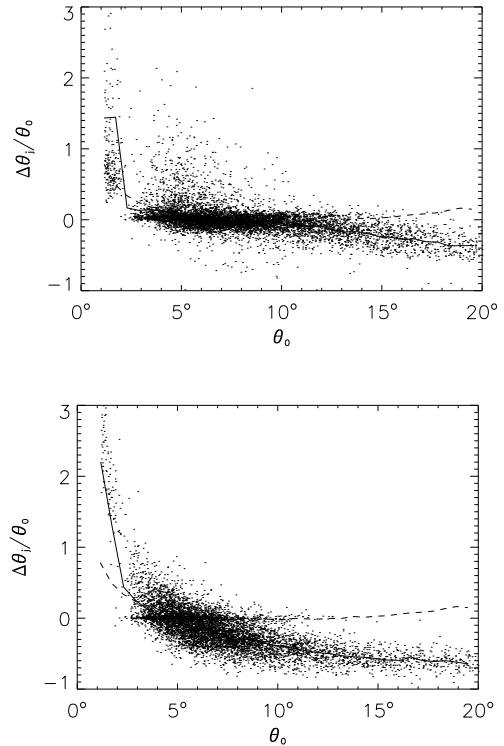


FIG. 3.— The correlation between θ_0 and $\Delta\theta_j/\theta_0$ for smooth fits (top) and sharp fits (bottom). In these scatter plots, the solid curves show a binned average of the data. Also shown is the binned average calculated from $t_{j,\Gamma}$ (dashed curve).

angles are systematically overestimated and wider ones underestimated. This is more evident with the sharp than the smooth fits. When the jet opening angle is used to correct the isotropic energy for beaming, this will lead to a clustering of the derived energy values.

To check the validity of the approximations used in deriving equations (3) and (4) we determined the characteristic time $t_{j,\Gamma}$ when θ equals $1/\Gamma$ in our model calculations. We then estimated the opening angle by setting t_j equal to $t_{j,\Gamma}$ in equation (4). This should give correct results in each case if these approximations are valid. The resulting angle, θ_j , is then compared to the known model opening angle, θ_0 , in a similar way as the for the power law fits. Figure 3 also shows the binned average of the correlation between $\Delta\theta_j/\theta_0$ and θ_0 for θ_j calculated from $t_{j,\Gamma}$ (dashed curve). It clearly shows that the approximations used in deriving equations (3) and (4) break down for very narrow jets. This is because the jet break time can be very close to the deceleration time (Panaiteescu et al. 1998) where the approximations are not valid. These approximations also break down in very wide jets since these may not necessarily be assumed to be ultra relativistic at the time of the jet break. Figure 3 also shows that the correlation is stronger when θ_j is calculated from t_j (solid curves), so t_j is overestimated from the fits when the jet is narrow and underestimated in wide ones. One explanation is that the jet break time is not necessarily within the limited time range used in the fits, but it could also be due to ν_m crossing the observing frequency, especially in narrow jets. In the latter

case, those events can be identified from a small value of the smoothness factor, n , because the break around ν_m is much smoother than the jet break. This crossing of ν_m through the observing frequency is also the cause of the small wings in the distribution of $\Delta p_1/p$ and $\Delta p_2/p$ seen around the value -0.1 in figure 2. By looking at the individual spectra, we can identify those events where ν_m or ν_c cross the observing frequency. We find that the latter case does not have a significant effect on our results.

3. CONCLUSION

We have shown that results from the standard procedure of fitting afterglow light curves with a broken power law must be interpreted with caution. There can be systematic differences in the evaluation of the electron energy distribution index from the slope of the fitted curves and a strong correlation between the relative difference in the opening angle estimated from the light curve break time and the initial opening angle of the jet. These findings are partly due to the approximations used in deriving equations (1)-(4) being used out of their validity limits. This applies to approximations in both the dynamical (Bianco & Ruffini 2005) and the radiation properties of the expanding shell. These differences and particularly the correlation is also a consequence of difficulties in accurately determining the jet break time from the afterglow light curves.

The fitting procedure we used was completely automatic and often did not converge. In some cases it resulted in erroneous parameter values and we therefore adopted a threshold on the reduced χ^2 to eliminate those bad fits. To test the effect of this threshold, we removed it from the selection criteria and re-examined the data, still considering only those events where the light curve break time was within the time range of our data. This left us with over 90% of the original population. The most significant changes we find in the results, were slightly larger standard deviations in the relative difference distributions and more spread in the $\Delta\theta_j/\theta_0$ - θ_0 correlation thereby weakening it. The spike around 0 seen in the $\Delta\theta_j/\theta_0$ distribution for the sharp fits in figure 2 (diamonds) also becomes a dominating feature. This indicates that by using the χ^2 as the strongest filter, we remove the bad fits from the ensemble but may also lose some useful fits.

It is known theoretically that the magnitude of the jet break, $\Delta\alpha = \alpha_2 - \alpha_1$, can be used to differentiate between a wind like or a constant density environment (e.g. Panaitescu et al. 1998). The systematic differences we find in the evaluation of p indicate that $\Delta\alpha$ is under-

estimated in most of our events. Similar conclusion is obtained for a population of afterglow light curves in a wind medium, the main difference being that p_1 did not show a systematic deviation in that case. This renders the method of using $\Delta\alpha$ to distinguish between density profiles impractical.

The correlation between the opening angle estimate and the known opening angle puts a strong limit on interpretations of the beaming corrected energy of the burst (e.g. Frail et al. 2001; Ghirlanda et al. 2004; Friedman & Bloom 2005). The clustering of the jet break time due to limited time span of the data together with a generous use of the approximations used in determining θ_j results in a bias towards moderate opening angles, approximately between 2° - 10° . The rather large standard deviation in the $\Delta\theta_j/\theta_0$ distribution also makes the results unreliable. Using the beaming corrected energy as a basis for cosmological studies therefore calls for a very careful determination of t_j and θ_j .

The synthetic burst population studied in this paper is computer generated and the power law fit should in theory be perfect. The fact that the differences between the parameters derived from the fits and the known model parameters are so significant, makes the accuracy of power law fits to real measurements of afterglow light curves a concern. In real bursts, effects such as density fluctuations and energy injection, can change the shape of afterglow light curves as may for example be the case for GRB 021004 (e.g. Lazzati et al. 2002) and GRB 030329 (e.g. Sheth et al. 2003). These were densely sampled and were not well fitted with a broken power law due to bumps in the light curves. It is not hard to see that grainier measurements of the same events could have been fitted with a broken power law, leading to even larger uncertainties in the parameter estimates than discussed here. It should also be noted that other models are capable of explaining light curve breaks, the most popular being the structured jet model (Rossi et al. 2002). There the light curve break depends on the observer's viewing angle rather than the jet opening angle. Hence the methodology adopted in this letter is not directly applicable to that model.

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